EXPLANATIONS

58. (A) Let *x* be the number of inches representing 1 foot. Set up a proportion and solve for *x*:

$$\frac{x}{1} = \frac{0.125}{125}$$

x = 0.001 in.

59. (B) Divide the rate (55 miles per hour) by the number of seconds in an hour (60 minutes x 60 seconds = 3,600 seconds):

 $\frac{55}{3,600}$ miles per second

Multiply by the number of feet in a mile (5, 280):

 $\frac{55 \times 5,280}{3,600}$ feet per second

60. (**-4**)

4w = 2w - 82w = -8w = -4

61. (45) Let x = number of students with only cats as pets. Let y = number of students with only dogs as pets.

Calculate *x* and *y* using the given information:

There are 20 students who have cats, and of those 20 students, 3 have both cats and dogs.

If 3 out of 20 students also have dogs, then x = 20 - 3 = 17.

There are 23 students who have dogs, and of those 23 students, 3 have both cats and dogs.

If 3 out of 23 students also have cats, then y = 23 - 3 = 20.

To find the total number of students surveyed, add the number of students who **only** have cats (x), the number of students who **only** have dogs (y), the number of students who have both (3), and the number of students who have neither (5):

3 + 5 + x + y = 8 + 17 + 20 = 45

62. (D) Since both ratios have *y* in common, solve for *x* and *z* in terms of *y* in both equations.

Using y:x = 1:4, solve for x in terms of y:

$$\frac{x}{y} = \frac{1}{4}$$
$$x = \frac{1}{4}y$$

Using the ratio y:z = 4:5, solve for z in terms of y:

$$\frac{y}{z} = \frac{4}{5}$$
$$z = \frac{5}{4}y$$

The question states x + y + z = 50. Substitute from the two equations above and solve for *y*:

$$\frac{1}{4}y + y + \frac{5}{4}y = 50$$
$$\frac{10}{4}y = 50$$
$$10y = 200$$
$$y = 20$$

63. (**B**)
$$2k = m + 3$$
, so $k = \frac{m + 3}{2}$.

Substitute each value of *m* to find the values of *k*:

$$k = \frac{5+3}{2} = \frac{8}{2} = 4$$

$$k = \frac{7+3}{2} = \frac{10}{2} = 5$$

$$k = \frac{9+3}{2} = \frac{12}{2} = 6$$

The set *k* is {4, 5, 6}.

64. (A)
$$7 + 3n + 6 - 4n - 8 =$$

(7 + 6 - 8) + (3n - 4n) =
5 - n

65. (A) The sum of Adrianna's course grades equals 4 times the mean (average) of her grades:

 $90 \times 4 = 360$

Roberto has the same sum (360) as Adrianna. Find the mean of his course grades:

 $360 \div 5 = 72$

66. Liam originally has twice as many stamps as Kevin, so the equation is L = 2K.

After Liam gives 8 stamps to Kevin, the equation becomes L - 8 = (K + 8) + 12, which simplifies to L - 8 = K + 20.

Substitute L = 2K into the equation: 2K - 8 = K + 20.

Solve for *K*:

2K - K = 20 + 8

K = 28

Substitute to find L: $L = 2K = 2 \times 28 = 56$.

So, Liam started with 56 stamps.

67. (162) To find angle the *x*, first find the measure of angle PQR by finding the measure of angle PSR:

 $m \angle PSR = m \angle PQR$ $m \angle PSR = 180 - 72$ $m \angle PSR = 108$

The measure of angle PQR is also 108.

Find the measure of angle *x*: 108 + 90 + x = 360 198 + x = 360x = 162 **68.** Add the three expressions: 5 - n + 3(2n + 7) + (-2)(2.5n + 4).

Distribute 3 and -2: 5 - n + 6n + 21 - 5n - 8.

Combine the variable terms and combine the constants:

(-n + 6n - 5n) + (5 + 21 - 8) = 18.

69. (B) Line segment \overline{RS} is the altitude, or height, of triangle QRP. The length of \overline{QP} is 8 cm.

Use the information to find the area of triangle QRP:

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24$$
 sq cm.

There are 4 congruent triangles in the pyramid, so the surface area of the pyramid **excluding** the base is $4 \times 24 = 96$ sq cm.

70. (B) Let 2x = the width and 3x = the length.

Draw the rectangle to help visualize.



Since 2 times width + 2 times length = perimeter:

$$2(2x) + 2(3x) = 510$$

 $4x + 6x = 510$
 $10x = 510$
 $x = 51$
 $2x = 102 \text{ cm and } 3x = 153 \text{ cm}$

71. (D) Multiply each term by 2 to eliminate the fraction, and isolate *x*:

$$-4(2) < \left(\frac{x}{2}\right)(2) < 2(2)$$

 $-8 < x < 4$

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Therefore, x must be between -8 and 4.
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72. (63) If x is the smaller consecutive integer, then x + 1 is the larger consecutive integer. Use their sum (-15) to find x:

x + (x + 1) = -15 2x + 1 = -15 2x = -16x = -8

The two consecutive integers are -8 and -7.

One is added to the smaller integer: -8 + 1 = -7, and 2 is subtracted from the larger integer: -7 - 2 = -9.

Find the product: $-7 \times -9 = 63$

73. (B) Let *x* be the total number of colored pencils in the box.

Set up a proportion to find *x*:

$$\frac{2}{7} = \frac{6}{x}$$
$$2x = 42$$
$$x = 21$$

If there are 6 red pencils, then the number of pencils that are **not** red is 21 - 6 = 15.

74. (B) Use proportions to make the conversions:

Lorgs to dollars:

$$\frac{140}{x} = \frac{7}{1} \\ 7x = 140 \\ x = $20$$

Dalts to dollars:

 $\frac{16}{x} = \frac{0.5}{1}$ 0.5x = 16x = \$32

Total dollars = 20 + 32 = \$52

75. (B) The shaded region is a right triangle. Each leg is 1 unit in length.

So the area is $A = \frac{1}{2}bh = \frac{1}{2}(1)(1) = \frac{1}{2}$ or 0.5 square units.

76. To create an expression equivalent to |(-8) - (-3) + 6| - |6 - 11|:

Step 1: Simplify the first absolute value term.

(-8) - (-3) = -8 + 3 = -5

-5 + 6 = 1

Simplify the second absolute value term.

6 - 11 = -5

Therefore the equivalent expression is

- |1| |-5|.
- **77. (A)** Let *x* be the price per pound of the meat. Set up an equation to show what Mrs. Cranston spent:

5(0.90) + 8x = 26.904.50 + 8x = 26.908x = 22.40x = 2.80

The price per pound of the meat was \$2.80.

78. (A) The probability that both cards are **not** blue is the same as the probability that both cards **are** red.

There are 4 red cards out of the 10, so the probability of the first card being red is $\frac{4}{10}$.

Now there are 9 cards left, and 3 of those are red, so the probability of the second card being red is $\frac{3}{9}$.

Multiply the two probabilities to find the probability that both cards are red (**not** blue):

$$\frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$$

79. (**D**) 1 sind = 4 lorgs, so 1 sind > 1 lorg.

2 harps = 5 sinds, so 1 harp > 1 sind.

1 plunk = 3 harps, so 1 plunk > 1 harp, meaning that 1 plunk > 1 sind and 1 lorg.

2 plunks = 5 dalts, so 1 plunk > 1 dalt.

Therefore, the plunk is the most valuable.

80. (B) Let x be the number of second-, third-, and fourth-year students. Then the total number of students in the college is 663 + x.

Set up a proportion and solve for *x*:

 $\frac{15}{1} = \frac{663 + x}{179}$ 663 + x = 179(15) 663 + x = 2,685 x = 2,022

81. (B) According to the chart, 22% of people walk to work and 4% ride a bicycle.

Subtract to find the percentage of how many more people walk than bicycle:

 $22\%^{-}4\% = 18\%$

To find the exact number of people, multiply 18% (0.18) by the number of people working in Center City (15,000):

 $15,000 \times 0.18 = 2,700$

82. (B) To find the smallest factor of 91, list the factors: 1, 7, 13, and 91.

The smallest factor (other than 1) is 7.

Of the options listed (30, 35, 39, and 44), **only** 35 is a multiple of 7.

83. (D) Let *x* be the remaining side of the actual banner.

Set up a proportion:

 $\frac{x}{16} = \frac{36}{12}$ x = 48 ft

84. (C) For each row, multiply the number of students by the score.

Add the products together and divide by the total number of students to find the mean (average) of the 10 students:

$$\frac{85(4) + 75(4) + 65(2)}{10} = \frac{340 + 300 + 130}{10}$$
$$= \frac{770}{10} = 77$$

85. (C) The first integer is *l*, so the second is l + 1, the third is l + 2, then l + 3, and finally l + 4.

Since g is the fifth and greatest of the integers, g = l + 4.

Substitute I + 4 for g and simplify:

$$\frac{l+g}{2} = \frac{l+l+4}{2} = \frac{2l+4}{2} = l+2$$

86. (99) Let *x* be the number of oak trees when 264 pine trees are planted.

Set up a proportion and solve for x:

$$\frac{x}{264} = \frac{3}{8}$$
$$8x = 762$$
$$x = 99$$

87. (D) Set up an equation to express Tien's age (*T*) and Jordan's age (*J*) today:

$$T = \frac{1}{4}J$$

Two years from now, Tien's age will be T + 2 and Jordan's age will be J + 2. Set up an equation about the relationship between Tien's age and Jordan's age in two years:

$$T + 2 = \frac{1}{3}(J + 2)$$

Solve the above equation for *T*:

$$T = \frac{1}{3}(J + 2) - 2$$

Now set the two equations equal to each other and solve for *J*:

$$\frac{1}{4}J = \frac{1}{3}(J + 2) - 2$$

$$\frac{1}{4}J = \frac{1}{3}J - \frac{4}{3}$$

$$-\frac{1}{12}J = -\frac{4}{3}$$

$$J = -\frac{4}{3}\left(-\frac{12}{1}\right)$$

$$J = 16$$

88. Solution:

Answer for second equation: -4

Answer for third equation: 28

Answer for fourth equation: -56

24 = -0.5(x + 8)

Distribute the -0.5 on the right side of the equation: 24 = -0.5x - 4.

Add 4 to both sides of the equation to eliminate the -4 on the right side: 28 = -0.5x.

Divide both sides by -0.5: -56 = x.

89. (D) $2\frac{1}{5} + 3\frac{3}{10} + 4\frac{2}{5} + 5\frac{1}{2}$

Convert all the fractions to a common denominator (10):

$$2\frac{2}{10} + 3\frac{3}{10} + 4\frac{4}{10} + 5\frac{5}{10}$$

= (2 + 3 + 4 + 5) + $\left(\frac{2 + 3 + 4 + 5}{10}\right)$
= 14 + $1\frac{4}{10} = 15\frac{2}{5}$

90. (C) The length of the stick must be the greatest common factor of 72 and 30. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. Of those, **only** 1, 2, 3, and 6 are also factors of 72. The greatest of these is 6.

91. (B) Create a list of the possible pairs. Let the cookies be named A, B, C, D, E, and F.

AB, AC, AD, AE, AF BC, BD, BE, BF CD, CE, CF

DE, DF

EF

There are a total of 15 possible pairs of cookies that Aiden can choose.

92. (C) Set up proportions to figure out how many slides Deion and Kyra can create in 1 hour:

Deion

 $\frac{5}{20} = \frac{x}{60}$ 20x = 300x = 15

Deion can create 15 slides in 1 hour.

Kyra

 $\frac{3}{10} = \frac{x}{60}$ 10x = 180x = 18

Kyra can create 18 slides in 1 hour.

Add the two rates to figure out how many slides they can create together in 1 hour:

$$15 + 18 = 33$$

93. (C) Since LN = $\frac{1}{8}$, point N is located at $4\frac{5}{16} + \frac{1}{8} = 4\frac{7}{16}$.

So M must be between point L, $4\frac{5}{16}$, and point N, $4\frac{7}{16}$.

Point L can also be written as 4.3125, and point N can be written as 4.4375.

The **only** option given that lies between those two points is 4.35.

94. Solution:

Start with 6x = 9x + 13.

Subtract 9x from both sides to move all terms involving x to one side.

6x - 9x = 13

Combine like terms.

$$-3x = 13$$

Divide both sides by -3 to solve for *x*.

$$x = -\frac{13}{3}$$
$$x = -4\frac{1}{3}$$

95. (B) Ryan has 130 pages left to read (150 - 20). He read 20 pages in 30 minutes, which means he read at a rate of 40 pages per 1 hour. To find out how much longer it will take him to finish the assignment, divide the total number of pages remaining (130) by the number of pages he is able to read per hour (40):

$$\frac{130}{40} = 3\frac{1}{4}$$

96. (C) It is easier to rewrite $\frac{M}{N}$ as $M \div N$ since they are both fractions.

$$M \div N = \frac{W}{x} \div \frac{y}{z} = \frac{W}{x} \times \frac{z}{y} = \frac{Wz}{xy}$$

97. (B) The question asks for integers from 12 to 30 that are **not** divisible by 2 or 3.

The set of consecutive integers is {12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}.

Since all even numbers are divisible by 2, eliminate all even numbers, leaving the odd numbers in the set: {13, 15, 17, 19, 21, 23, 25, 27, 29}.

Eliminate those integers that are multiples of 3 (15, 21, and 27). The remaining integers are: {13, 17, 19, 23, 25, 29}. herefore, there are 6 numbers in the set that are multiples of **neither** 2 nor 3.

98. (B) Since 3n is even, then 3n + 1 must be odd. If 3n + 1 is odd, then 3n + 3 and 3n + 5 are also odd. So there are a total of 3 numbers in this range that are odd.

99. Solution:

There are three correct points: (1, 3), (3, 9), and (4, 12).

Find the cost per notebook: $\frac{12}{4} = 3$.

The equation of the line is y = 3x, where y is the total cost of x notebooks.

Check each point by plugging the *x*-value into the equation y = 3x. If the *y*-value from the equation matches the *y*-value of the point, then the point is on the line y = 3x.

Check all seven points:

(1, 1) y = 3(1) = 3 y-values do not match; incorrect point

(1, 3) y = 3(1) = 3 y-values match; CORRECT point

(2, 5) y = 3(2) = 6 y-values do not match; incorrect point

(3, 7) y = 3(3) = 9 y-values do not match; incorrect point

(3, 9) y = 3(3) = 9 y-values match; CORRECT point

(4, 12) y = 3(4) = 12 y-values match; CORRECT point

(5, 14) y = 3(5) = 15 y-values do not match; incorrect point

100. (**D**) Solve the equation for *z*:

$$\frac{w}{x} = \frac{y}{z}$$
$$wz = xy$$
$$z = \frac{xy}{w}$$

101. (C) Convert the ratios into fractions of WZ. Use the sum of the ratios for the denominator.

WX:XY:YZ = 4:2:3
WX =
$$\frac{4}{4 + 2 + 3} = \frac{4}{9}$$

XY = $\frac{2}{4 + 2 + 3} = \frac{2}{9}$

The part of WZ that is WY is the sum of those fractions:

$$WY = \frac{4}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$$

Find the length of WZ:

$$WZ = 8 - (-10) = 18$$

The value of WY is $\frac{2}{3}(18) = 12$.

102. (C) Find 1% of 0.02: 0.02 × $\frac{1}{100}$ = 0.0002

The greatest allowable thickness would be 0.02 + 0.0002 = 0.0202 inch.

103. (D) Calculate the highest score for each section by adding the lowest score to the range:

Section I: 65 + 28 = 93

Section II:
$$62 + 25 = 87$$

Section III: 67 + 22 = 89

The overall highest score is 93, and the overall lowest score is 62.

Subtract the lowest score from the highest score to find the overall range:

93 - 62 = 31

104. (C) Take each city's number of schools and multiply by the number of students. It is not necessary to calculate all 5 of these. Cities M and N have the same number of students, so just calculate the number of students in City M because it has more schools than City N. The same goes for Q and R — only Q needs to be calculated because it has more schools than R.

 $M = 8 \times 500 = 4,000$ $P = 9 \times 400 = 3,600$ $Q = 6 \times 700 = 4,200$

City Q has the greatest number of students.

105. (**D**) There are 6 digits in the repeating decimal (769230), so 7 would be the first, seventh, thirteenth digit and so on. To find the 391st digit, divide 391 by 6.

 $391 \div 6 = 65 \text{ R1}$

Since the remainder is 1, that means the 391st digit is the same as the 1st digit, which is 7.

106. Solution:



Solve the inequality for *x*, then graph the solution on the number line:

 $-3 \leq 2x - 5$

Add 5 to both sides of the inequality: $2 \leq 2x$

Divide both sides by 2: $1 \leq x$

Rewrite the inequality with x on the left side: $x \ge 1$

The graph of $x \ge 1$ is a closed ray (solid point) beginning at 1 and extending to the right on the number line.

107.

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(D) 100(2 + 0.1)^2 - 100 = 100(2.1^2) - 100
= 100(4.41) - 100 = 441 - 100 = 341
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108. (C) The total number of handballs in the container is 4 + 5 + 8 + 9 + 11 = 37.

Since there are 8 yellow handballs, the probability of selecting a yellow handball is $\frac{8}{37}$.

109. (A) Each chair costs Leon \$150 to make, and he sells the chair for \$275. His profit is found by subtracting the cost from the price:

275 - 150 = 125 per chair

If Leon makes and sells 25 chairs in a week, his initial profit is $25 \times \$125 = \$3,125$. However, Leon has additional fixed expenses of \$1,250 per week, so this cost must also be subtracted to arrive at the profit.

His final profit is 3,125 - 1,250 = 1,875.

110. (D) Convert 4 ft 7 in. to inches.

Since 12 in. = 1 ft

4(12) + 7 = 55 inches

Multiply that by the conversion:

2.54 cm = 1 in. $55 \times 2.54 = 139.70 \text{ cm}$

111. (C) Find the location of J by using $JK = 3\frac{1}{2}$:

$$\frac{3}{8} - J = 3\frac{1}{2}$$
$$J = \frac{3}{8} - 3\frac{1}{2} = -3\frac{1}{8}$$

Find the location of M by using $JM = 9\frac{3}{4}$:

$$M - \left(-3\frac{1}{8}\right) = 9\frac{3}{4}$$
$$M + 3\frac{1}{8} = 9\frac{3}{4}$$
$$M = 9\frac{3}{4} - 3\frac{1}{8} = 6\frac{5}{8}$$

Use LM =
$$1\frac{1}{8}$$
 to find the location of L:
 $6\frac{5}{8} - L = 1\frac{1}{8}$
 $L = 6\frac{5}{8} - 1\frac{1}{8} = 5\frac{4}{8} = 5\frac{1}{2}$

112. (C)

$$4x - 3y = 12$$

$$4x = 3y + 12$$

$$x = \frac{3}{4}y + \frac{12}{4}$$

$$x = \frac{3}{4} + 3$$

113. (A) Determine the total number of servings of fruits and vegetables that the students ate by multiplying the number of servings by the number of students in each row of the table. Then add that column to get the total number of servings:

Number of Servings of Fruits and Vegetables	Number of Students	Number of Servings × Number of Students
0	5	0
1	7	7
2	3	6
3	4	12
4	0	0
5	1	5
		Total: 30

Calculate the mean by dividing the total number of servings of fruits and vegetables by the total number of students:

$$\frac{30}{20} = 1\frac{1}{2}$$

114. Solution:

<u>7</u> 5

Substitute b = 3, c = 4, and d = 5 in $\frac{\frac{c}{d}}{\frac{c}{b}} + \frac{c}{d}$: $\frac{\frac{4}{5}}{\frac{4}{3}} + \frac{4}{5}$.

Then simplify the complex fraction by multiplying the complex fraction by a form of 1, $\begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix}: \quad \frac{\frac{4}{5}}{\frac{4}{3}} + \frac{4}{5} = \frac{\frac{4}{5}}{\frac{4}{3}} \begin{pmatrix} \frac{3}{4} \\ \frac{3}{4} \end{pmatrix} + \frac{4}{5}.$ Simplify after multiplying by a form of 1: $\frac{\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)}{\left(\frac{4}{3}\right)\left(\frac{3}{4}\right)} + \frac{4}{5} = \frac{3}{\frac{5}{1}} + \frac{4}{5}.$

Then simplify again and add the two fractions: $\frac{\frac{3}{5}}{1} + \frac{4}{5} = \frac{3}{5} + \frac{4}{5} = \frac{7}{5}$.